Main Ideas:

- Analyze conditional statements
- Write the converse of conditional statements

A *conditional statement* is a statement (true/false) that can be written in ***if-then*** form.

Example 1: If a polygon has 7 sides, then it is a heptagon.

If you live in Thousand Oaks, then you live in Ventura County.

Example 2: Use the advertisement below to write a statement in if-then form.

Get a **free** phone
when you sign up
for a 2-year contract!

____

____

Example 3: Write this statement in if-then form: September has 30 days.

____

Example 4: Write in if-then form: Points are collinear if they lie on the same line.

____

Write an if-then statement of your own: ____

Hypothesis: the phrase that follows the word ***if***

Conclusion: the phrase that follows the word ***then***

Example 5: Underline the hypothesis once, the conclusion twice, and tell if each statement is true/false.

A) If two angles are supplementary, then they add up to 180°.

B) If you mix blue and yellow, you get red.

C) A polygon is a decagon if it has ten sides.

D) If two angles are adjacent, then they form a linear pair.

E) Two angles are congruent, if they have the same measure.

F) If two angles are congruent, then they are vertical angles.

KEY CONCEPT ***Related Conditionals***

Statement	Formed by	Symbols	Examples
Conditional	given hypothesis and conclusion	$p \to q$	If two angles have the same measure, then they are congruent.
Converse	exchanging the hypothesis and conclusion of the conditional	$q \to p$	If two angles are congruent, then they have the same measure.

A _______________ is a statement where the ______ and _________ of the conditional statement are _________.

For example, "If two numbers are both even, then their sum is even" is a true statement.
The converse would be "If the sum of two numbers is even, then the numbers are even," which is not a true statement.

Example 6: Write the converse of each statement and tell if it's true or false.

A) Conditional ($p \to q$): If it is August, then the month has 31 days. **True/False?**

Converse ($q \to p$): ____ **True/False?**

B) Conditional ($p \to q$): If an angle has a measure of 40°, then it is acute. **True/False?**

Converse ($q \to p$): ____ **True/False?**

C) Conditional: If a person is from South America, then he speaks Spanish. **True/False?**

Converse: ____ **True/False?**

D) Conditional: If two angles are supplementary, then they add up to 180°. **True/False?**

Converse: ____ **True/False?**

Example 7: Rewrite the following implications as a conditional statement, and indicate whether the conditional statement is true or false. Then, rewrite the conditional as a converse, and indicate whether the converse is true or false. Explain.

A) An angle that measures 95° is an obtuse angle.

Conditional: ____ **True/False?**

Converse: ____ **True/False?**

B) Complementary angles add up to 90°.

Conditional: ____ **True/False?**

Converse: ____ **True/False?**

C) A triangle is a polygon.

Conditional: ____ **True/False?**

Converse: ____ **True/False?**

D) An even number is divisible by two.

Conditional: ____ **True/False?**

Converse: ____ **True/False?**

Conclusion: Can you assume that if a conditional statement is true, its converse is true?__________

Can you assume that if a conditional statement is false, its converse is false? ________

Main Ideas:

- Identify and use basic postulates about points, lines, and planes.
- Learning the basic structure of a proof.

In geometry, a **postulate** or **axiom** is a statement that is accepted as true. Postulates describe fundamental relationships in geometry.

Postulate: Through any two points, there is exactly one line.

Postulate: Through any three points not on the same line, there is exactly one plane.

Postulate: A line contains at least two points.

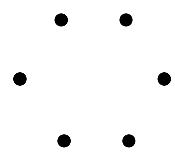
Postulate: A plane contains at least three points not on the same line.

Postulate: If two points lie in a plane, then the line containing those points lies in the plane.

Postulate: If two lines intersect, then their intersection is exactly one point.

Postulate: If two planes intersect, then their intersection is a line.

Ex 1: Some snow crystals are shaped like regular hexagons. How many lines must be drawn to interconnect all vertices of a hexagonal snow crystal? ________



Ex 2: Determine if each statement is **always**, **sometimes**, or **never** true. Explain your answer.

1. If there is a line, then it contains at least 3 noncollinear points.

2. If there are points A, B, and C, then there is exactly one plane that contains them.

3. If there are two lines, then they will intersect.

4. If points J and K lie in plane *M*, then $\overleftrightarrow{JK}$ lies in *M*.

A **proof** is a logical argument in which each statement you make is supported by a statement that is accepted as true.

There are different types of proof formats: **paragraph**, **flow-proof**, and **2-column**. In this class, we will only use 2-column proofs.

Every 2-column proof has these parts:

- ☑ Given (this is the information that is provided to you)
- ☑ Prove (this is what you are trying to show is true based on the given information)
- ☑ Column of statements (these are the statements that make up your logical argument)
- ☑ Column of reasons (these are the supporting statements that “back up” your argument)

Ex 3: Given: Points H and I Prove: $\overleftrightarrow{HI}$ is unique

Statements	**Reasons**
1. Points H and I exist	1. Given
2. $\overleftrightarrow{HI}$ is unique	2. Postulate: Through any two points, there is exactly one line

The reasons in a proof will be definitions, postulates, theorems, and algebraic properties.

Here is a list of some definitions and theorems that you have already learned which will be used frequently in geometric proofs. These should be memorized because they frequently come up in writing proofs.

Definition of a right angle: If an angle is a right angle, then its measure is _____.

Definition of complementary angles: If two angles are complementary, then their sum is ____.

Definition of supplementary angles: If two angles are supplementary, then their sum is ______.

Supplements Theorem: If two angles form a linear pair, then they are ______________________.

Vertical Angles Theorem: If two angles are vertical, then they are ______________________.

Definition of perpendicular lines: If two lines are perpendicular, then they form ______ angles.

Definition of Congruency: If $\overline{QR} \cong \overline{ST}$, then ____________________

Definition of a Midpoint: If M is the midpoint of $\overline{AB}$, then _________________

Segment Addition Postulate: If K lies between J and L, then _____________________________

Midpoint Theorem: If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

Below is a proof of the Midpoint Theorem…

Given: M is the midpoint of $\overline{AB}$ **Prove:** $\overline{AM} \cong \overline{MB}$

Statements	**Reasons**
1.	1.
2.	2.
3.	3.

Main Idea: Using algebraic properties to write a two-column proof

CONCEPT SUMMARY *Properties of Real Numbers*

The following properties are true for any numbers a, b, and c.

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	If $a = b$, then $a \cdot c = b \cdot c$ and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

(page 111 in textbook)

Example 1: State the property that justifies each statement.

1. If w + 5 = 9, then w = 4. ____________________
2. If 10m = 50, then m = 5. ____________________
3. If c = a and a = t, then c = t. ________________
4. If x = 9, then 9 = x. ______________________
5. z = z ____________________________________
6. If 3(x – 4) = 6, then 3x – 12 = 6 _____________
7. If x = 8, then 3x + 2 = 3(8) + 2 ______________
8. If x = 7 and y = 7, then x = y. _______________

Two Column Proofs

In Geometry, we use two column proofs to prove step-by-step that something is true. It's like a lawyer developing a case using logical arguments based on evidence to lead the jury to a conclusion favorable to their case. This is a form of deductive reasoning.

We will start with algebraic proofs to get you used to the format and process of building a proof.

Example 2: Given: $\frac{7x+3}{4} = 6$ Prove: x = 3

Statement	Reason
1. $\frac{7x+3}{4} = 6$	1.
2. $7x+3=24$	2.
3. $7x=21$	3.
4. x = 3	4.

Example 3: Given: 3(x – 2) = 42 Prove: x = 16

Statement	Reason
1. 3(x – 2) = 42	1.
2. 3x – 6 = 42	2.
3. 3x = 48	3.
4. x = 16	4.

Example 4: Given: 2(5 – 3a) – 4(a + 7) = 92 Prove: a = -11

Match each statement with the correct reason.

Statement	Answer	Reason
1. 2(5 – 3a) – 4(a + 7) = 92		A. Distributive Property
2. 10 – 6a – 4a – 28 = 92		B. Division Property
3. -10a – 18 = 92		C. Addition Property
4. -10a = 110		D. Given
5. a = -11		E. Simplify (combine like terms)

Example 5: Write a two-column proof to show that: If $3\left(x - \frac{5}{3}\right) = 1$, then x = 2.

<u>Step-by-Step Instructions for Writing Two-Column Proofs</u>

1. Read the problem over carefully. Write down the information that is given to you because it will help you begin the problem. Also, make note of the conclusion to be proved because that is the final step of your proof. This step helps reinforce what the problem is asking you to do and gives you the first and last steps of your proof.

2. Draw an illustration of the problem to help you visualize what is given and what you want to prove. Oftentimes, a diagram has already been drawn for you, but if not, make sure you draw an accurate illustration of the problem. **Include marks** that will help you see congruent angles, congruent segments, parallel lines, or other important details if necessary**. Mark the information given and the conclusions** that can be made based on what is given and based on definitions, postulates and theorems that exist.

3. Use the information given to help you deduce the preliminary steps of your proof. Every step must be shown, regardless of how trivial it appears to be. Beginning your proof with a good first step is essential to arriving at a correct conclusion.

4. Use the conclusion, or argument to be proven, to help guide the statements you make. Remember to support your statements with reasons, which can include definitions, postulates, or theorems.

5. Once you have arrived at your solution, you may choose to **read through the two-column proof you've written** to be assured that each step has a reason. This helps emphasize the clarity and effectiveness of your argument.

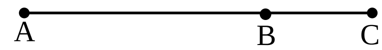
The steps above will help guide you through the rest of the geometry sections you encounter. While they may seem painful and frustrating at times, two-column proofs are extremely helpful because they break things down that seem trivial or intuitive into steps that answer the question "why."

Main Ideas:

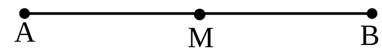
- Use definitions, properties, and theorems to write two-column proofs involving segments

Review of Segments : Complete each statement below.

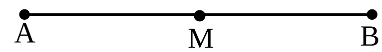
Segment Addition Postulate
If B is between A and C, then ____________________________.



Definition of a Midpoint
If M is the midpoint of $\overline{AB}$, then ________________________.



Midpoint Theorem
If M is the midpoint of $\overline{AB}$, then ________________________.



Def. of Congruency
If AB = XY, then ____________________

Reflexive Property	**Symmetric Property**	**Transitive Property**
AB = AB	If AB = CD, then CD = AB	If AB = CD and CD = EF, then AB = EF
$\overline{AB} \cong \overline{AB}$	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$

Example 1: Given: $\overline{JK} \cong \overline{QT}$; JK = 3x + 5; QT = 2x + 8
Prove: x = 3

Draw a sketch:

Statement	Reason
1. $\overline{JK} \cong \overline{QT}$; JK = 3x + 5; QT = 2x + 8	1.
2. JK = QT	2.
3. 3x + 5 = 2x + 8	3.
4. x + 5 = 8	4.
5. x = 3	5.

Example 2: Given: M is the midpoint of $\overline{OG}$; OM = x + 4; MG = 5(3x-2)

Prove: x = 1

Draw a sketch:

Statement	Answers	Reason
1. M is the midpoint of $\overline{OG}$; OM = x + 4; MG = 5(3x-2)		A. Substitution
2. OM = MG		B. Subtraction Prop.
3. x + 4 = 5(3x-2)		C. Division Prop.
4. x + 4 = 15x – 10		D. Given
5. -14x + 4 = -10		E. Distributive Prop.
6. -14x = -14		F. Subtraction Prop.
7. x = 1		G. Definition of a midpoint

Example 3: Given: $\overline{HI} \cong \overline{LO}$; $\overline{LO} \cong \overline{YA}$; $\overline{YA} \cong \overline{KU}$ Prove: $\overline{HI} \cong \overline{KU}$

Draw a sketch:

Statement	Reason
1. $\overline{HI} \cong \overline{LO}$; $\overline{LO} \cong \overline{YA}$	1.
2.	2. Transitive Prop.
3. $\overline{YA} \cong \overline{KU}$	3.
4.	4.

Example 4: Given: PQ = RS Prove: PR = QS

Statement	Reason
1. PQ = RS	1.
2. PQ + QR = RS + QR	2.
3. PQ + QR = PR RS + QR = QS	3.
4. PR = QS	4.

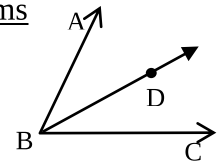
Main Ideas:

- Use definitions, properties, and theorems to write two-column proofs involving angles

Important Angle Definitions, Postulates and Theorems

Def. of an Angle Bisector: If $\overrightarrow{BD}$ bisects ∠ABC,

then ________ ≅ ________



Def. of Supplementary Angles: If ∠X and ∠Y are supplementary, then _______ + _______ = _____

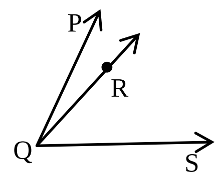
Def. of Complementary Angles: If ∠X and ∠Y are complementary, then _______ + _______ = _____

Def. of a Right Angle: If ∠K is a right angle, then _________ = ______

Def. of Congruency: If ∠P ≅ ∠D, then ________ = ________

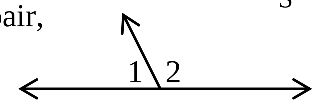
Angle Addition Postulate: If R is in the interior of ∠PQS,

then ________ + ________ = ________



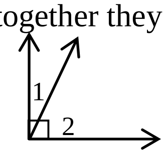
Supplement Theorem: If ∠1 and ∠2 form a linear pair,

then they are ________________________



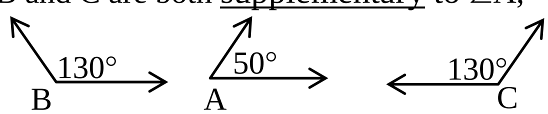
Complement Theorem: If ∠1 and ∠2 are adjacent and together they form a right angle,

then they are ______________________________.



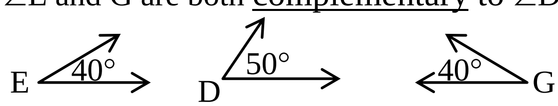
Congruent Supplements Theorem: If ∠B and C are both <u>supplementary</u> to ∠A,

then ______________________



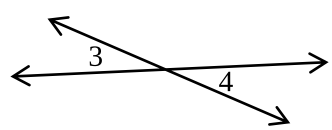
Congruent Complements Theorem: If ∠E and G are both <u>complementary</u> to ∠D,

then ______________________



Vertical Angles Theorem: If ∠3 and ∠4 are vertical,

then ________________



Perpendicular Lines and Right Angles

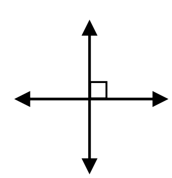
Definition of perpendicular lines: If two lines are perpendicular, then they form right angles.

Theorem: All right angles are congruent.

Theorem: Perpendicular lines form congruent adjacent angles.

Theorem: If 2 angles are congruent and supplementary, then they are both right angles.

Theorem: If two congruent angles form a linear pair, then they are both right angles.



Reflexive Property	**Symmetric Property**	**Transitive Property**
m∠A = m∠A	If m∠A = m∠B, then m∠B = m∠A	If m∠A = m∠B and m∠B = m∠C, then m∠A = m∠C
∠A ≅ ∠A	If ∠A ≅ ∠B, then ∠B ≅ ∠A	If ∠A ≅ ∠B and ∠B ≅ ∠C, then ∠A ≅ ∠C

Example 1: **Given**: R in the interior of ∠PQS;
m∠PQS = 70°; m∠PQR = (14x – 44)°; m∠RQS = 5x°
Prove: x = 6

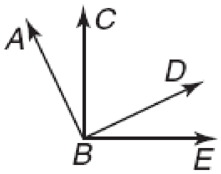
Sketch:

Statement	Answer	Reason
1. R in the interior of ∠PQS; m∠PQS = 70°; m∠PQR = (14x – 44)°; m∠RQS = 5x°		A. Substitution
2. m∠PQR + m∠RQS = m∠PQS		B. Simplify
3. (14x – 44) + 5x = 70		C. Division Prop.
4. 19x – 44 = 70		D. Given
5. 19x = 119		E. Addition Prop.
6. x = 6		F. Angle Addition Postulate

Example 2: **Given**: ∠O and ∠K are supplementary **Prove**: x = 25
m∠O = (4x + 10)°; m∠K = (3x – 5)°

Statement	Reason
1. ∠O and ∠K are supplementary m∠O = (4x + 10)°; m∠K = (3x – 5)°	1.
2. m∠O + m∠K = 180°	2.
3. (4x + 10) + (3x – 5) = 180	3.
4. 7x + 5 = 180	4.
5. 7x = 175	5.
6. x = 25	6.

Example 3: **Given**: ∠ABC and ∠CBD are complementary
∠DBE and ∠CBD form a right angle **Prove**: ∠ABC ≅ ∠DBE



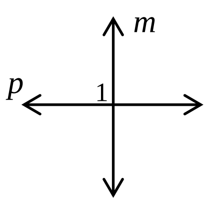
Statement	Reason
1.	1.
2. ∠DBE and ∠CBD are complementary	2.
3.	3.

Example 4: **Given:** $\overrightarrow{AT}$ bisects $\angle SAX$;
$m\angle SAT = (6x - 4)$; $m\angle TAX = (2x + 28)$ **Prove:** $x = 8$

Sketch:

Statement	Reason
1.	1.
2.	2. Definition of an angle bisector
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Example 5: **Given:** $p \perp m$ **Prove:** $x = 16$
$m\angle 1 = (4x + 26)°$



Statement	Reason
1. $p \perp m$	1. Given
2. ______ is a right angle	2.
3. $m\angle 1$ = ______	3.
4.	4.
5.	5.
6.	6.