Objectives: To find the probability of a simple event, and apply the Fundamental Counting Principle to find the possible number of outcomes of compound events.

<u>What is probability?</u>

- Probability is the measure of chance. It describes how likely it is that an event will occur.
- We can define the probability of a ***simple event*** as a ratio of the number of favorable outcomes for the event to the total number of possible outcomes of the event.
- The probability of an event *a* can be expressed as: $P(a) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$
- The ratio has to be reduced.
- The outcomes can be represented using set notation. The possible outcomes are called a "sample space", usually represented by "S"; and the favorable outcomes are called a "subset", or an "event". Example: S= {H, T}; A={H}

<u>Finding the number of outcomes</u>

<u>For simple events</u>: count the outcomes

Examples: Write the number of outcomes, and list them.

1) One Die- ___ outcomes

2) One deck of cards- ___ outcomes

3) One coin- ___ outcomes

4) One fair number cube- ___ outcomes

Example: Find the probability, and reduce the ratio, if possible.

5) P(rolling a 3)= $\frac{\text{number of ways of rolling a 3}}{\text{Total number of outcomes}}$ =

Practice:

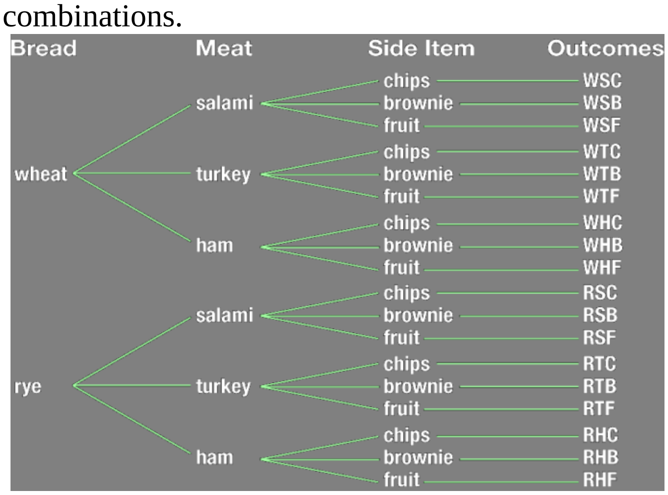
1) You have a bag with four blue marbles, and three red marbles. If you were to draw a marble from the bag without looking, what is the probability that you will select a blue marble?

<u>For compound events (more than one event):</u>

The total outcomes of each event are found by using a tree diagram, or by using the Fundamental Counting Principle. The **Fundamental Counting Principle** says that if event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by the event N can occur in m·n ways.

Example:

At football games, a student concession stand sells sandwiches on either wheat or rye bread. The sandwiches come with salami, turkey, or ham, and either chips, a brownie, or fruit. Find the number of possible sandwich combinations.



Using the Fundamental Counting Principle:

bread x meat x side

______ x ______ x ______ = ______ outcomes

The counting principle can be used for both independent and dependent events. If the outcome of one event does not affect the outcome of another event and vice versa, the events are called **independent events**.

Example:
For the Breakfast Special at the Country Pantry, customers can choose their eggs scrambled, fried, or poached, whole wheat or white toast, and either orange, apple, tomato, or grapefruit juice. How many different Breakfast Specials can a customer order?
Solution: A customer's choice of eggs does not affect his or her choice of toast or juice, so the events are independent. There are 3 ways to choose eggs, 2 ways to choose toast, and 4 ways to choose juice. By the Fundamental Counting Principle, there are $3 \cdot 2 \cdot 4$ or 24 ways to choose the Breakfast Special.

Practice:
2) A restaurant serves 5 main dishes, 3 salads, and 4 desserts. How many different meals could be ordered if each has a main dish, a salad, and a dessert?

3) Marissa brought 8 T-shirts and 6 pairs of shorts to summer camp. How many different outfits consisting of a T-shirt and a pair of shorts does she have?

4) How many license plate numbers consisting of three letters followed by three numbers are possible when repetition is allowed?

If the outcome of an event *does* affect the outcome of another event, the two events are said to be **dependent events**. The Fundamental Counting Principle still applies.

Example:
The guests at a sleepover brought 8 videos. They decided they would only watch 3 videos. How many orders of 3 different videos are possible?
Solution: After the group chooses to watch a video, they will not choose to watch it again, so the choices of videos are dependent events. There are 8 choices for the first video. That leaves 7 choices for the second. After they choose the first 2 videos, there are 6 remaining choices. Thus, by the Fundamental Counting Principle, there are $8 \cdot 7 \cdot 6$ or 336 orders of 3 different videos.

Practice:
5) Three students are scheduled to give oral reports on Monday. In how many ways can their presentations be ordered?

6) How many license plates consisting of three letters followed by three numbers are possible when no repetition is allowed?

7) Sixteen teams are competing in a soccer match. Gold, silver, and bronze medals will be awarded to the top three finishers. In how many ways can the medals be awarded?

Objectives: Determine whether an arrangement is a permutation or combination, and find the number of possibilities for the given situation.

Permutations

When a group of objects or people are arranged in a certain order, the arrangement is called a _______________.

Permutations	The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.
Permutations with Repetitions	The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

The rule for permutations with repetitions can be extended to any number of objects that are repeated.

Example 1: *From a list of 20 books, each student must choose 4 books for book reports. The first report is a traditional book report; the second, a poster; the third, a newspaper interview with one of the characters; and the fourth, a timeline of the plot. How many different orderings of books can be chosen?*

Solution: Since each book report has a different format, order is important. You must find the number of permutations of 20 objects taken 4 at a time. You can find the solution to a permutation by:

1) using the formula:

$$P(n, r) = \frac{n!}{(n-r)!} \quad \text{Permutation formula}$$

$$P(20, 4) = \frac{20!}{(20-4)!} \quad n = 20,\ r = 4$$

$$= \frac{20!}{16!} \quad \text{Simplify.}$$

$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16}^{1} \cdot \cancel{15}^{1} \cdot \ldots \cdot \cancel{1}^{1}}{\cancel{16}_{1} \cdot \cancel{15}_{1} \cdot \ldots \cdot \cancel{1}_{1}} \quad \text{Divide by common factors.}$$

$$= 116{,}280$$

2) using the Fundamental Counting Principle. You just need to know how many possible outcomes there still exist after each event has occurred. For the problem above, we have

____ · ____· ____· _____ = ____________ outcomes

3) using your calculator:

P(20, 4) = ___ nPr ____= ________

Example 2:

The high school chorus has been practicing 12 songs, but there is time for only 5 of them at the spring concert. How many different orderings of 5 songs are possible?

Practice:

1) P(6, 3)

2) How many different ways can the letters in MONDAY be arranged?

3) The top 5 runners at the cross-country meet will receive trophies. If there are 22 runners in the race, in how many ways can the trophies be awarded?

Combinations

An arrangement or selection of objects in which order is *not* important is called a ________________.

Combinations	The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n-r)!r!}$.

Example 3: *How many groups of 4 students can be selected from a class of 20?*
Solution: Since the order of choosing the students is not important, you must find the number of combinations of 20 students taken 4 at a time. Combinations can be solved by:

1) using the formula

$$C(n, r) = \frac{n!}{(n-r)!r!} \qquad \text{Combination formula}$$

$$C(20, 4) = \frac{20!}{(20-4)!4!} \qquad n = 20, r = 4$$

$$= \frac{20!}{16!4!} \text{ or } 4845$$

There are 4845 possible ways to choose 4 students.

2) using your calculator

C(20, 4) = __ nCr __ = ____

When we are combining (or choosing from) more than one group, we need to multiply each combination.

Example 4: *In how many ways can you choose 1 vowel and 2 consonants from a set of 26 letter tiles? (Assume there are 5 vowels and 21 consonants.)*

C(5, 1): One of 5 vowels are drawn.
C(21, 2): Two of 21 consonants are drawn.
C(5,1) · C(21, 2) = ______ · ______ = ______

$$C(5, 1) \cdot C(21, 2) = \frac{5!}{(5-1)!1!} \cdot \frac{21!}{(21-2)!2!}$$

$$= \frac{5!}{4!} \cdot \frac{21!}{19!2!}$$

$$= 5 \cdot 210 \text{ or } 1050$$

Practice:

4) Evaluate C(10, 5)

5) From a standard deck of 52 cards, in how many ways can 5 cards be drawn?

6) From a group of 10 men and 12 women, how many committees of 5 men and 6 women can be formed?

More practice:

In the following problems, determine whether each situation involves a permutation or a combination. Then find the number of possibilities.

7) the first-, second-, and third-place finishers in a race with 8 contestants

8) an arrangement of the letters in the word *cube*.

Objectives: Determine whether a set of events is either independent or dependent, and find the probability for the given situation.

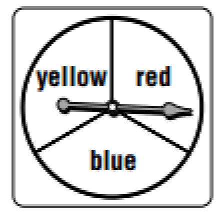
Recall that, if the outcome of one event does not affect the outcome of another event and vice versa, the events are called ___________________. If the outcome of an event *does* affect the outcome of another event, the two events are said to be ________________.

Probability of Independent Events

Probability of Two Independent Events	If two events, A and B, are independent, then the probability of both occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

- Another way to represent that is $P(A \cap B) = P(A) \cdot P(B)$. (In set notation, the "∩" is the intersection symbol, which is the equivalent of "and". $P(A \cap B)$ is read "the probability of A and B". Whenever you see "∩" or "and", remember you need to MULTIPLY.)
- In many problems with independent events, you will notice that an item involved in the problem is replaced. This is also known as "**with replacement**".
- You can use the Fundamental Counting Principle when finding the probability of independent events.

Example 1:
In a board game, each player has 3 different-colored markers. To move around the board, the player first spins a spinner to determine which piece can be moved. He or she then rolls a die to determine how many spaces that colored piece should move. On a given turn, what is the probability that a player will be able to move the yellow piece more than 2 spaces?



Example 2:
A box contains 5 triangles, 6 circles, and 4 squares. If a figure is removed, replaced, and a second figure is picked, what is the probability that a triangle and then a circle will be picked?

Practice:
1) A die is rolled 3 times. Find the probability of each event.
A) a 1 is rolled, then a 2, then a 3

B) a 1 or a 2 is rolled, then a 3, then a 5 or a 6

C) 2 odd numbers are rolled, then a 6

D) a number less than 3 is rolled, then a 3, then a number greater than 3

2) A bag contains 5 red marbles and 4 white marbles. A marble is selected from the bag, then replaced, and a second selection is made. What is the probability of selecting 2 red marbles?

3) A jar contains 7 lemon jawbreakers, 3 cherry jawbreakers, and 8 rainbow jawbreakers. What is the probability of selecting 2 lemon jawbreakers in succession providing the jawbreaker drawn first is then replaced before the second is drawn?

Probability of Dependent Events

Probability of Two Dependent Events	If two events, *A* and *B*, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

- This is also expressed as $P(A \cap B) = P(A) \cdot P(B|A)$ (read as "probability of B given A). A is the first event, and B is the second event.
- In many of the problems dealing with dependent events, you will notice that the items involved are *not* replaced; this is also known as "**without replacement**".
- You can also use the Fundamental Counting Principle in finding the probability of dependent events; you just have to make sure that you have the correct probability for each event, keeping into consideration what the new total outcomes are for each event.

Example 3: There are 7 dimes and 9 pennies in a wallet. Suppose two coins are to be selected at random, without replacing the first one. Find the probability of picking a penny and then a dime.

Example 4: What is the probability of drawing, without replacement, 3 hearts, then a spade from a standard deck of cards?

Practice: Find each probability.

4) The cup on Sophie's desk holds 4 red pens and 7 black pens. What is the probability of her selecting first a black pen, then a red one?

5) What is the probability of drawing two cards showing odd numbers from a set of cards that show the first 20 counting numbers if the first card is not replaced before the second is chosen?

6) There are 3 quarters, 4 dimes, and 7 nickels in a change purse. Suppose 3 coins are selected without replacement. What is the probability of selecting a quarter, then a dime, and then a nickel?

7) A basket contains 4 plums, 6 peaches, and 5 oranges. What is the probability of picking 2 oranges, then a peach if 3 pieces of fruit are selected at random?

8) A photographer has taken 8 black and white photographs and 10 color photographs for a brochure. If 4 photographs are selected at random, what is the probability of picking first 2 black and white photographs, then 2 color photographs?

Objectives: Determine whether a set of events is mutually exclusive or inclusive, and find the probability of the given situation.

Mutually Exclusive Events

Events that cannot occur at the same time are called ___________________________ events.

Probability of Mutually Exclusive Events	If two events, A and B, are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

- P(AUB) = P(A) + P(B) (In set notation, the "U" is a union symbol, which is equivalent to "or". P(AUB) is read "the probability of A or B". Whenever you see "U" or "OR", remember you need to ADD.)
- This formula can be extended to any number of mutually exclusive events.

Example 1: *To choose an afternoon activity, summer campers pull slips of paper out of a hat. Today there are 25 slips for a nature walk, 35 slips for swimming, and 30 slips for arts and crafts. What is the probability that a camper will pull a slip for a nature walk or for swimming?*

Solution: These are mutually exclusive events. Note that there is a total of 90 slips.

$$P(\text{nature walk or swimming}) = P(\text{nature walk}) + P(\text{swimming})$$
$$= \frac{25}{90} + \frac{35}{90} \text{ or } \frac{2}{3}$$

The probability of a camper's pulling out a slip for a nature walk or for swimming is $\frac{2}{3}$.

Example 2: *By the time one tent of 6 campers gets to the front of the line, there are only 10 nature walk slips and 15 swimming slips left. What is the probability that more than 4 of the 6 campers will choose a swimming slip?*

Solution:

$$P(\text{more than 4 swimmers}) = P(5 \text{ swimmers}) + P(6 \text{ swimmers})$$
$$= \frac{C(10, 1) \cdot C(15, 5)}{C(25, 6)} + \frac{C(10, 0) \cdot C(15, 6)}{C(25, 6)}$$
$$\approx 0.2$$

The probability of more than 4 of the campers swimming is about 0.2.

Practice: Find each probability.

1) A bag contains 45 dyed eggs: 15 yellow, 12 green, and 18 red. What is the probability of selecting a green or a red egg?

2) The letters from the words LOVE and LIVE are placed on cards and put in a box. What is the probability of selecting an L or an O from the box?

3) A pair of dice is rolled, and the two numbers are added. What is the probability that the sum is either a 5 or a 7?

4) An art box contains 12 colored pencils and 20 pastels. If 5 drawing implements are chosen at random, what is the probability that at least 4 of them are pastels?

<u>Inclusive Events</u>

Two events that could occur at the same time are called _______________ events.

Probability of Inclusive Events	If two events, A and B, are inclusive, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

- This formula could also be expressed as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Notice that you need to add both events separately, but then subtract the inclusive events.

Example 3: What is the probability of drawing a face card or a black card from a standard deck of cards?

Solution: The two events are inclusive, since a card can be both a face card and a black card.

$$\begin{aligned} P(\text{face card or black card}) &= P(\text{face card}) + P(\text{black card}) - P(\text{black face card}) \\ &= \frac{3}{13} + \frac{1}{2} - \frac{3}{26} \\ &= \frac{8}{13} \text{ or about } 0.62 \end{aligned}$$

The probability of drawing either a face card or a black card is about 0.62

Example 4: The letters of the alphabet are placed in a bag. What is the probability of selecting a vowel or one of the letters from the word QUIZ?

Practice: Find each probability

5) What is the probability of drawing a red card or an ace from a standard deck of cards?

6) Three cards are selected from a standard deck of 52 cards. What is the probability of selecting a king, a queen, or a red card?

7) A pair of dice is rolled. What is the probability that the sum is odd or a multiple of 3?

8) The Venn diagram at the right shows the number of juniors on varsity sports teams at Elmwood High School. Some athletes are on varsity teams for one season only, some athletes for two seasons, and some for all three seasons. If a varsity athlete is chosen at random from the junior class, what is the probability that he or she plays a fall or winter sport?