

Objective: To analyze two-way tables, and find the probability of given events.

A **conditional probability** is a probability in which a certain prerequisite condition has already been met.

Example 1: Suppose we want to know the probability that our next card will be a face card given that the first card was the 7 of diamonds. The formal notation for this is $P(\text{Face}|7\spadesuit)$. This is read as "The probability of a face card given that we already have been dealt the 7 of diamonds".

Solution: Often times the math for these situations is very logical. In our case, we have simply reduced the deck by one card and there are still 12 face cards in the deck. Therefore $P(\text{Face}|7\spadesuit) = \frac{12}{51} = \frac{4}{17} \approx 0.24$.

Example 2: Two cards are dealt from a standard deck of 52 cards. Find the conditional probability.
 $P(\text{2nd red}|\text{1st } 2\clubsuit)$

Practice:

1) $P(\text{2nd red}|\text{1st } 2\heartsuit)$

2) $P(\text{2nd } \spadesuit|\text{1st red})$

Another way we can look at conditional probabilities is through the use of **two-way tables** or **contingency tables**. These are often referred to as two-way tables because there are two distinct pieces of information gathered in these tables. For example, we may record how many siblings you have and in how many activities you participate in school. Two-way tables can be filled in either using counts or probabilities.

Example 3

Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Suppose we randomly select one student.

	Bus	Walk	Car	Other
9 th or 10 th grade	106	30	70	4
11 th or 12 th grade	41	58	184	7

- What is the probability that the student walked to school?
- What is the probability that the student was a 9th or 10th grader?
- What is the probability that a student either rode the bus or is in 11th or 12th grade?

Solutions: In order to calculate any probability, you must **first add up all the rows and columns** so that you know the number of possible outcomes for each category. Then, you can set up the probability of the event.

	Bus	Walk	Car	Other	Total
9 th or 10 th grade	106	30	70	4	210
11 th or 12 th grade	41	58	184	7	290
Total	147	88	254	11	500

a) There were 88 walkers out of 500 total students or $\frac{88}{500} = \frac{22}{125} \approx 0.18$.

b) There were 210 9th or 10th graders out of 500 total students or $\frac{210}{500} = \frac{21}{50} = 0.42$.

c) There are 147 kids who rode the bus and there are 290 kids who are 11th or 12th graders. However, notice that these two categories intersect and we must be careful not to count the 41 kids who are in both categories twice. We will take the 290 11th or 12th graders and just add the 106 bus riders who are not 11th and 12th graders for a total of 396 students. The probability of selecting an 11th or 12th grader or a bus rider

is $\frac{396}{500} = \frac{99}{125} \approx 0.79$.

In the example above, note that the total across the bottom, $147+88+254+11$, and the total for the last column, $210+290$, both add up to 500. This is true of all 2-way tables.

Example 4

Consider the completed chart in the solution of part a) of Example 3.

a) What is the probability that a student is in 11th or 12th grade given that they rode in a car to school?

b) What is $P(\text{Walk}|\text{9th or 10th grade})$?

Example 5

The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a contingency table that tracks the type of customer and type of ice cream.

Practice

Felipe surveyed students at his school. He found that 78 students own a cell phone and 57 of those students own an MP3 player. There are 13 students that do not own a cell phone, but own an MP3 player. Nine students do not own either device. Construct a two-way table summarizing the data.

Example 6

A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

Practice

Suppose that 40% of adults like eating bananas while 60% like eating apples. Suppose also that 32% of adults like eating both. What is the conditional probability that a randomly selected adult likes apples given that they like bananas? Use a Venn Diagram to answer this question.